연습문제 해답

9장

연습문제 9.1

1. $dr/dt = k/r$ **3.** $dv/dt = k(M - v)$

5. $dy/dt = k(N - y)$ **7.** 예 **9.** 아니오 **11.** 예

15. (a) $\frac{1}{2}$, -1 **17.** (d)

19. (a) 0이거나 감소해야 한다

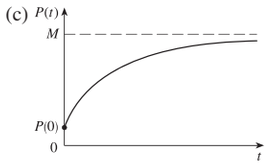
(c) $y = 0$ (d) $y = 1/(x + 2)$

21. (a) $0 < P < 4200$ (b) $P > 4200$

(c) $P = 0$, $P = 4200$

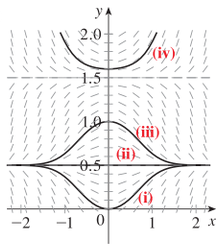
25. (a) III (b) I (c) IV (d) II

27. (a) 처음에; 양의 상태를 유지하지만 감소

(c) 

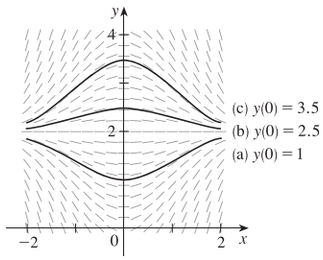
29. c가 c_s에 접근함에 따라 0에 접근한다.

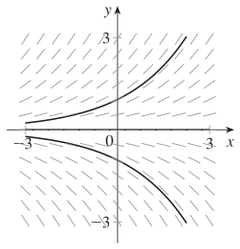
연습문제 9.2

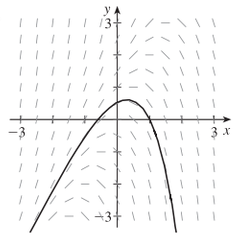
1. (a) 

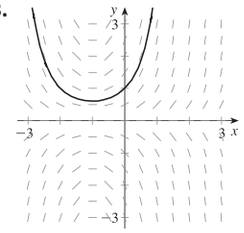
(b) $y = 0.5$, $y = 1.5$

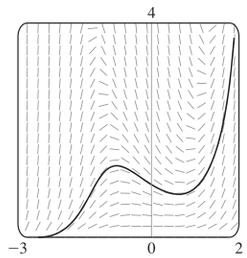
3. III 5. IV

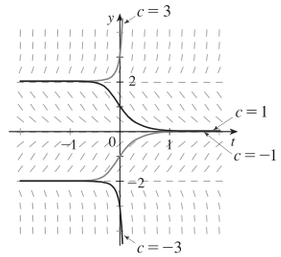
7. 

9. 

11. 

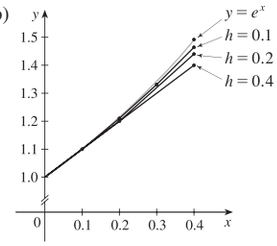
13. 

15. 

17. 

$-2 \leqslant c \leqslant 2$; -2, 0, 2

19. (a) (i) 1.4 (ii) 1.44 (iii) 1.4641

(b) 

과소 추정

(c) (i) 0.0918 (ii) 0.0518 (iii) 0.0277

오차도 (대략) 반으로 줄어든 것으로 보인다.

21. -1, -3, -6.5, -12.25 **23.** 1.7616

25. (a) (i) 3 (ii) 2.3928 (iii) 2.3701 (iv) 2.3681

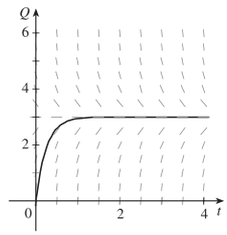
(c) (i) -0.6321 (ii) -0.0249 (iii) -0.0022 (iv) -0.0002

오차도 (대략) 10으로 나눈 것으로 보인다.

27. (a), (d) (b) 3

(c) 예, $Q = 3$

(e) 2.77 C



연습문제 9.3

1. $y = -1/(x^3 + C),\ y = 0$ **3.** $y = (\frac{1}{4}x^2 + C)^2,\ y = 0$

5. $y = \pm\sqrt{x^2 + 2\ln|x| + C}$

7. $e^y - y = 2x + \sin x + C$ **9.** $p = Ke^{(t^3/3)-t} - 1$

11. $\theta \sin\theta + \cos\theta = -\frac{1}{2}e^{-t^2} + C$

13. $y = -\ln(1 - \frac{1}{2}x^2)$ **15.** $A = b^3e^{b\sin br}$

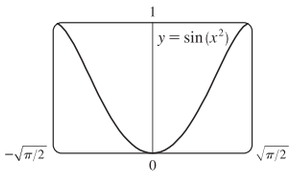
17. $u = -\sqrt{t^2 + \tan t + 25}$

19. $\frac{1}{2}y^2 + \frac{1}{3}(3 + y^2)^{3/2} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + \frac{41}{12}$

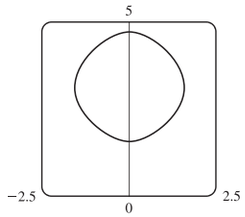
21. $y = \sqrt{x^2 + 4}$ **23.** $y = Ke^x - x - 1$

25. (a) $\sin^{-1}y = x^2 + C$

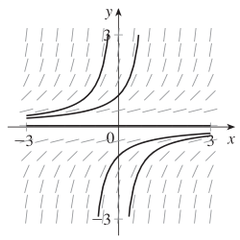
(b) $y = \sin(x^2),\ -\sqrt{\pi/2} \leqslant x \leqslant \sqrt{\pi/2}$ (c) 아니오



27. $\cos y = \cos x - 1$

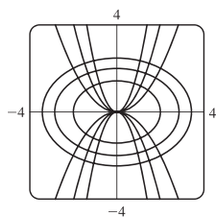


29. (a), (c)

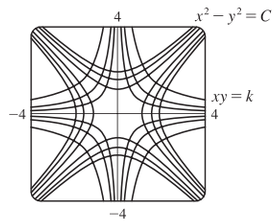


(b) $y = \dfrac{1}{K - x}$

31. $y = Cx^2$



33. $x^2 - y^2 = C$



35. $y = 1 + e^{2-(x^2/2)}$ **37.** $y = (\frac{1}{2}x^2 + 2)^2$

39. $Q(t) = 3 - 3e^{-4t};\ 3$ **41.** $P(t) = M - Me^{-kt};\ M$

43. (a) $x = a - \dfrac{4}{(kt + 2/\sqrt{a})^2}$

(b) $t = \dfrac{2}{k\sqrt{a-b}}\left(\tan^{-1}\sqrt{\dfrac{b}{a-b}} - \tan^{-1}\sqrt{\dfrac{b-x}{a-b}}\right)$

45. (a) $C(t) = (C_0 - r/k)e^{-kt} + r/k$

(b) r/k; 농도는 C_0값에 관계없이 r/k에 접근한다.

47. (a) $15e^{-t/100}$ kg (b) $15e^{-0.2} \approx 12.3$ kg

49. 약 4.9% **51.** g/k

53. (a) $L_1 = KL_2^k$ (b) $B = KV^{0.0794}$

55. (a) $dA/dt = k\sqrt{A}\,(M - A)$

(b) $AA(t) = M\left(\dfrac{Ce^{\sqrt{M}\,kt} - 1}{Ce^{\sqrt{M}\,kt} + 1}\right)^2$, 여기서 $C = \dfrac{\sqrt{M} + \sqrt{A_0}}{\sqrt{M} - \sqrt{A_0}}$이고

$A_0 = A(0)$

57. (b) $v_e = \sqrt{2gR}$ (c) $v_e \approx 11{,}173$ m/s ≈ 11.2 km/s

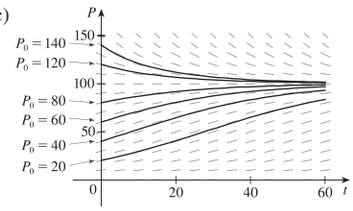
연습문제 9.4

1. (a) 1200; 0.04 (b) $P(t) = \dfrac{1200}{1 + 19e^{-0.04t}}$ (c) ≈87

3. (a) 100; 0.05

(b) P가 0 또는 100에 가까울 때 선 $P = 50$에서; $0 < P_0 < 100$; $P_0 > 100$

(c)



해들은 100으로 접근한다; 일부는 증가하고 일부는 감소하고 일부는 변곡점이 있지만 일부는 그렇지 않다; $P_0 = 20$ 및 $P_0 = 40$인 해는 $P_0 = 50$에서 변곡점을 갖는다.

(d) $P = 0$, $P = 100$; 다른 해는 $P = 0$에서 $P = 100$으로 이동한다.

5. (a) $\approx 3.23 \times 10^7$ kg (b) ≈1.55년 **7.** 9000

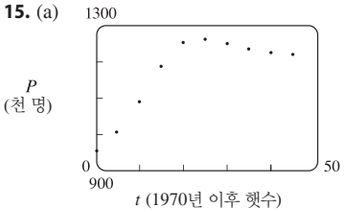
9. (a) $\dfrac{dP}{dt} = \dfrac{1}{305}P\left(1 - \dfrac{P}{20}\right)$ (b) 62억 4천만

(c) 75억 7천만; 138억 7천만

11. (a) $\dfrac{dy}{dt} = ky(1 - y)$ (b) $y = \dfrac{y_0}{y_0 + (1 - y_0)e^{-kt}}$

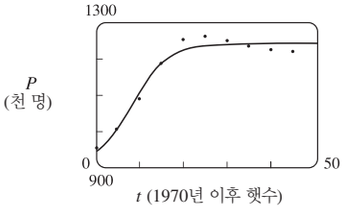
(c) 3:36 PM

15. (a)



(b) $f(t) = \dfrac{345.5899}{1 + 7.9977e^{-0.2482t}}$

(c) $P(t) = 900 + \dfrac{345.5899}{1 + 7.9977e^{-0.2482t}}$



(d) 인구가 124만 6천 명에 접근

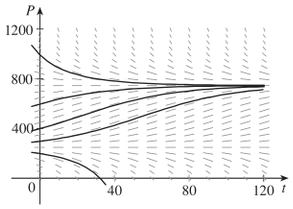
17. (a) $P(t) = \dfrac{m}{k} + \left(P_0 - \dfrac{m}{k}\right)e^{kt}$ (b) $m < kP_0$

(c) $m = kP_0$, $m > kP_0$ (d) 감소

19. (a) 물고기는 일주일에 15마리씩 잡힌다.

(b) (d) 참조 (c) $P = 250$, $P = 750$

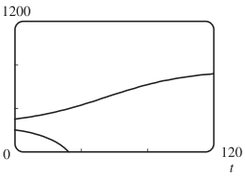
(d)



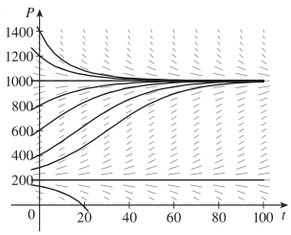
$0 < P_0 < 250$: $P \to 0$;
$P_0 = 250$: $P \to 250$;
$P_0 > 250$: $P \to 750$

(e) $P(t) = \dfrac{250 - 750ke^{t/25}}{1 - ke^{t/25}}$

여기서 $k = \frac{1}{11}, -\frac{1}{9}$



21. (b)



$0 < P_0 < 200$: $P \to 0$;
$P_0 = 200$: $P \to 200$;
$P_0 > 200$: $P \to 1000$;
$P = 200$,
$P = 1000$

(c) $P(t) = \dfrac{m(M - P_0) + M(P_0 - m)e^{(M-m)(k/M)t}}{M - P_0 + (P_0 - m)e^{(M-m)(k/M)t}}$

23. (a) $P(t) = P_0 e^{(k/r)[\sin(rt - \phi) + \sin\phi]}$ (b) 존재하지 않는다.

연습문제 9.5

1. 아니오 **3.** 예; $\dfrac{du}{dt} - \dfrac{e^t}{\sqrt{t}}u = -\sqrt{t}$ 5. $y = 1 + Ce^{-x}$

7. $y = x - 1 + Ce^{-x}$ **9.** $y = \frac{2}{3}\sqrt{x} + C/x$

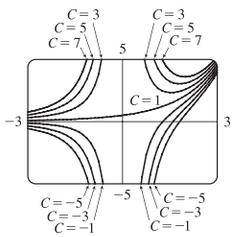
11. $y = x^2(\ln x + C)$ **13.** $y = \frac{1}{3}t^{-3}(1 + t^2)^{3/2} + Ct^{-3}$

15. $y = e^{-\sin x}\int xe^{\sin x}\,dx + Ce^{-\sin x}$ **17.** $y = x^2 + 3/x$

19. $y = \dfrac{1}{x}\ln x - \dfrac{1}{x} + \dfrac{3}{x^2}$ **21.** $u = -t^2 + t^3$

23. $y = -x\cos x - x$

25. $y = \dfrac{(x - 1)e^x + C}{x^2}$

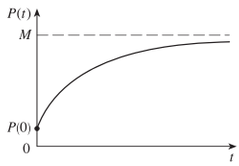


29. $y = \pm\left(Cx^4 + \dfrac{2}{5x}\right)^{-1/2}$

31. (a) $I(t) = 4 - 4e^{-5t}$ (b) $4 - 4e^{-1/2} \approx 1.57$ A

33. $Q(t) = 3(1 - e^{-4t})$, $I(t) = 12e^{-4t}$

35. $P(t) = M + Ce^{-kt}$



37. $y = \frac{2}{5}(100 + 2t) - 40{,}000(100 + 2t)^{-3/2}$; 0.2275 kg/L

39. (b) mg/c (c) $(mg/c)[t + (m/c)e^{-ct/m}] - m^2g/c^2$

41. (b) $P(t) = \dfrac{M}{1 + MCe^{-kt}}$

연습문제 9.6

1. (a) x = 포식자, y = 피식자; 증가는 피식자만을 먹는 포식자에 의해서만 제한된다.

(b) x = 먹이, y = 포식자; 증가는 포화 밀도와 먹이만을 먹는 포식자에 의해 제한된다.

3. (a) 경쟁

(b) (i) $x = 0$, $y = 0$: 인구가 0

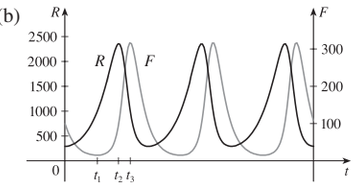
(ii) $x = 0$, $y = 400$: x인구가 없으면 y인구가 400으로 안정화된다.

(iii) $x = 125$, $y = 0$: y인구가 없으면 x인구가 125로 안정화된다.

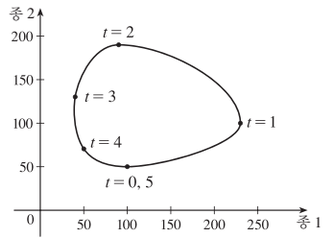
(iv) $x = 50$, $y = 300$: 두 개체군 모두 안정적이다.

5. (a) 토끼 개체수는 약 300에서 시작하여 2400으로 증가했다가 다시 300으로 감소한다. 여우 개체는 100에서 시작하여 약 20으로 감소하고 약 315로 증가하고 100으로 감소하고 주기가 다시 시작된다.

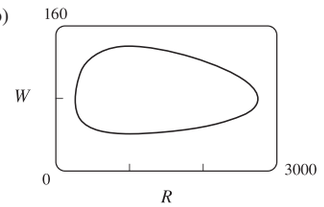
(b)



7.



9. (b)



11. (a) 개체수는 5000에서 안정화된다.

(b) (i) $W = 0$, $R = 0$: 개체수가 0

(ii) $W = 0$, $R = 5000$: 늑대가 없을 때 토끼 개체수는 항상 5000이다.

(iii) $W = 64$, $R = 1000$: 두 개체군 모두 안정적이다.

(c) 토끼 1000마리와 64마리로 안정화된다.

(d)