Errata: Advanced Calculus by Kim and Lee

Chapter 0 Preliminaries

Section 0.1.

(1) p.6: In Example 0.11, "which is a contraction" should be "which is a contradiction".

Section 0.3.

(1) p.13: In Line 2, "$A_1 \times A_2$ is onto" should be "$A_1 \times A_2$ is countable".

Section 0.4.

(1) p.15: In Exercise 0.43, "Then the following..." should be "Prove that the following...".

Section 0.5.

(1) p.16: In Example 0.48, "such" should be "such that".

(2) p.18: In Exercise 0.60, "iff" should be "if and only if".

Section 0.6.

(1) p.20: In Problem 0,3, "$k_1+k_2 \cdots +k_m$" should be "$k_1+k_2+\cdots+k_m$".

(2) p.22: In the end of Problem 0.14, ".(the period)" should be added.

Chapter 1 Sequences and Series

Section 1.1.

(1) p.26: In Example 1.3, "$n \geq \mathbb{N}$" should be "$n \geq N$".

(2) p.26: In Example 1.7, "S_{2^n}" should be "a_{2^n}".

Section 1.3.

(1) p.30: In line 22, "...a strictly increasing sequence in $\mathbb{N}$" should be "...a sequence in $\mathbb{N}$ such that $n_k < n_{k+1}$ for all $k \in \mathbb{N}$".

(2) p.31: In the proof of Theorem 1.21, "closed and bounded intervals in $\mathbb{R}$" should be "closed and bounded intervals".

Section 1.4.

(1) p.32: In line 7, "ia" should be "is".

Section 1.8.

(1) p.44: In line 2, "$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{2^2} + \cdots$" should be "$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$".

(2) p.44: In line 7, "$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{2^p} + \cdots$" should be "$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$".

(3) p.44-47: In Examples 1.58, 1.69, Exercise 1.65, Corollaries 1.67, and 1.68, the numbers p, q, c should be rational, because n^p has not been defined for irrational p.

(4) p.46: In the end of Exercise 1.66, ".(the period)" should be added.

(5) p.46: The proof of Theorem 1.67 can be shorten by taking $m = n$, because $s_n \leq s_{2^n}$.

(6) p.46: In the proof of Corollary 1.68, "Hence by Theorem 1.67 that" should be "Hence it follows from Theorem 1.67 that".

(7) p.51: To solve Exercise 1.78, one may need a hint like "To prove the monotonicity of a function, you may need to apply the derivative test in Chapter 3 (Theorem 3.38)." Or it should go to the problem section of Chapter 4.

Chapter 2 Limits and Continuity

Section 2.1.

(1) p.59: In Example 2.6, "$|q_n - a| < 1/n$, and $|r_n - a| < 1/n$" should be "$0 < |q_n - a| < 1/n$, and $0 < |r_n - a| < 1/n$".

(2) p.59, 60: In Theorems 2.5, 2.8, and 2.9, "L, L_1, L_2" should be specified as "real numbers".

Section 2.4.

(1) p.69: In Theorem 2.39, "Let f is" should be "Let f be" and the open interval "(a, b)" should be the closed interval "$[a, b]$". The proof should be modified accordingly.

Section 2.5.

(1) p.74: In Problem 2.20 (ii), "t_2x_n" should be "t_2x_2".

Chapter 3 Differentiation

Section 3.2.

(1) p.81: In Theorem 3.15 (i), (ii), the commas "," should be the periods ".".

(2) p.82: In the proof of Theorem 3.19, "by the product rule" should be "by Theorem 2.8 (ii)".

Section 3.3.

(1) p.88: In Theorem 3.29, "Every exponential functions" should be "Every exponential function".

Section 3.5.

(1) p.94: "there exists a $c \in (a, x_0)$" should be "there exists a $c \in (a, x)$".
(2) p.95: In Case 4 of the proof of Theorem 3.47, it should be pointed out that $A = \infty$ and $B = -\infty$ is impossible.

(3) p.95: In Exercise 3.49, "$A = \infty$ and $B = \pm\infty$" should be "$A = \infty$ and $B = \infty$".

(4) p.96: In Example 3.50, "$t = \ln x$" should be "$x = \ln t$".

(5) p.97: In Exercise 3.56 (ii), "exists" should be "exists in $\mathbb{R}$".

Section 3.6.

(1) p.98: In the proof of Theorem 3.57, "f'' exists" should be "h'' exists".

Section 3.7.

(1) p.101: In Problem 3.3, "$a, b > 0$" should be "$a \in \mathbb{R}$ and $b > 0$".

(2) p.106: In Problem 3.32, "an open interval I" should be "an unbounded interval $I = (a, \infty)$".

Chapter 4 Integration

Section 4.1.

(1) p.113: In the proof of Theorem 4.12, "$i = 1, 2, \ldots, n$" should be "$k = 1, 2, \ldots, n$".

Section 4.2.

(1) p.114: In the proof of Lemma 4.18 (i), "Problem 0.13" should be "Problem 0.10" and "E is a bounded subset" should be "E is a nonempty bounded subset".

(2) p.115: In line 13, ".(the period)" should be added.

(3) p.118: In Example 4.27, "$\underline{\int}_{-a}^{0}$" should be "$\int_{-a}^{0}$".

Section 4.3.

(1) p.119: In the last two lines, "$\leq$" should be "$<$" and "$<$" should be "$\leq$".

Section 4.4.

(1) p.122: In line 5, "$\|P\| \leq \delta$" could be "$\|P\| < \delta$".

(2) p.126: In line 7, "dt" should be added in an integral.

(3) p.129: In line 5, the period "." should be removed.

(4) p.131: In line 1, "$x \in [a, b]$" should be "$x \in [c, d]$". In the second-to-last line, "$\leq$" could be "$<$".

(5) p.132: In line 14, "$\cos x^2$" should be "$\cos^2 x$".

Section 4.5.

(1) p.134: In line 6, the equality "$=$" should be inserted.

(2) p.135: In line 13, "$x \to a$" should be "$c \to a$".

(3) p.136: In line 6, "$\int_1^\infty e^{-x}\, dx = 1$" should be "$\int_1^\infty e^{-x}\, dx = e^{-1}$".

(4) p.137-138: In Example 4.60, "$\Gamma(x)$ exists in $[0, \infty)$" should be "$\Gamma(x)$ exists in $[0, \infty]$". "$2(1 - e^{-M/s})$" should be "$2e^{-M/2}$".

(5) p.138: In Exercise 4.61 (iii), "$2^{n+1}n!$" should be "$2^{2n}n!$".

(6) p.139: In Example 4.63, "if and if only" should be "if and only if".

(7) p.139: Exercise 4.66 is quite difficult and should go to the problem section.

Section 4.6.

(1) p.140: In Problem 4.1 (i), "continuous on $[0, 1] \setminus E$" should be "continuous on $(0, 1] \setminus E$". Problem 4.2 is quite easy and so should be removed or an exercise.

(2) p.142: Problem 4.13 should be corrected as follows:
If $f : [a, b] \to \mathbb{R}$ is a continuous one-to-one function, prove that

$$\int_a^b f(x)\, dx + \int_{f(a)}^{f(b)} f(y)\, dy = bf(b) - af(a).$$

(3) p.144: In Problem 4.19, the function g should satisfy the same condition as f.

(4) p.146: In Problem 4.25, "$0 \leq d_{n+1} - d_n$" should be "$0 \leq d_n - d_{n+1}$".

Chapter 5

Section 5.1.

(1) p.147: The title "Double series" should be "Double Series".

(2) p.147: In line 6, "$|a_{mn} - s| < \varepsilon$" should be "$|a_{mn} - a| < \varepsilon$".

(3) p.149: In Theorem 5.5, "$\{S_k\}_{k=1}^{\infty}$" should be "$\{S_k\}_{k=0}^{\infty}$" and "$\bigcup_{k=1}^{\infty} S_k = \mathbb{N}_0^2$" should be "$\bigcup_{k=0}^{\infty} S_k = \mathbb{N}_0^2$". Moreover, italic "$(i), (ii)$" should be roman "(i), (ii)".

Section 5.5.

(1) p.168: The title "Taylor series" should be "Taylor Series".

Section 5.6.

(1) p.176: In Problem 5.3, an "$*$" should be added.

Chapter 6

Section 6.2.

(1) p.187: In the statement of Theorem 6.9, "(Bolzano-Weierstarss)" should be "(Bolzano-Weierstrass)". In the proof of Theorem 6.9, "$\phi_2(\phi_1(k))$" should be "$\phi_1(\phi_2(k))$" and "$\phi_n \circ \cdots \circ \phi_1$" should be "$\phi_1 \circ \cdots \circ \phi_n$"

(2) p.189: In Example 6.18, "$1/k \in E$" should be "$1/k \in E \setminus \{0\}$".

Section 6.3.

(1) p.193: After Exercise 6.41, add "*Hint for (iii)*: Find a sequence $\{\mathbf{y}_k\}$ in E converging to $\operatorname{dist}(\mathbf{x}, E)$, and then apply the Bolzano-Weierstrass theorem.".

(2) p.193: After Exercise 6.43, add "*Hint*: For $\mathbf{x} \in \mathbb{R}^n$ and $r > 0$, let $E_r = \left\{\mathbf{y} \in \mathbb{R}^n : (x_1 - y_1)^2/a_1^2 + \cdots + (x_n - y_n)^2/a_n^2 \leq r^2\right\}$. Show that if $x_1^2/a_1^2 + \cdots + x_n^2/a_n^2 = r^2 < 1$, then $E_r(\mathbf{x}) \subset E$.".

Section 6.4.

(1) p.197: In line 12, "$k \in E$" should be $k \in \mathbb{N}$".

Section 6.5.

(1) p.198: In the paragraph after Exercise 6.56, "$X \subset \mathbb{R}^n$" should be "X"

(2) p.199: In the last paragraph of the proof of Theorem 6.66, insert "$X = A \cup B$" and "$X \subset U \cup V$".

(3) p.199: Exercise 6.61 should be moved to after Example 6.58.

(4) p.199: In the definition of E in the proof of Theorem 6.62, "$[a, b]$" should be "$(a, b]$".

(5) p.200: In the last paragraph of the proof of Theorem 6.66, insert "$U \cap V = \varnothing$" and "$A \cap B = \varnothing$".

Section 6.6.

(1) p.202: In Problem 6.1, "Prove that there exists..." should be "Prove that if N is a norm on $\mathbb{R}^n$, then there exists...".

(2) p.203: In Problem 6.11, "Then for every..." should be "Prove that for every...".

Chapter 7

Section 7.1.

(1) p.208: In Theorem 7.3, "$\mathbf{a} \in \overline{E}$" should be "$\mathbf{a}$ is a limit point of E", "$E \setminus \{\mathbf{a}\}$" should be "$E$", and "$B_r(\mathbf{a}) \setminus \{\mathbf{a}\}$" should be "$B_r(\mathbf{a})$".

Section 7.3.

(1) p.213: In Theorem 7.18, "$E \subset \mathbb{R}^n$ is closed" should be "$E \subset \mathbb{R}^n$ is nonempty and closed".

(2) p.215: "$g'(x^*) - 1$" should be "$1 - g'(x^*)$".

Section 7.4.

(1) p.217: For later reference, we need to introduce the zero matrix denoted by $O_{m\times n}$ or simply O.

(2) p.220: In line 5, Add "(*why?*)" after "is invertible".

Section 7.5.

(1) p.222: The title "The Weierstrass approximation theorem" should be "The Weierstrass Approximation Theorem".

Section 7.6.

(1) p.228: In Problem 7.4, "$p > 1$ and $q > 1$" should be "p and q are positive numbers with $p + q > 2$".

(2) p.230: In Problem 7.14 (i), "$f^{-1}(-\infty, y)$" should be "$f^{-1}((-\infty, y])$".

(3) p.232: In Problem 7.27 (i), "k" should be "j".

(4) p.232: In Problem 7.27 (ii), "$|e^{\|T\|} - 1|$" should be "$e^{\|\mathbf{T}\|}$".

(5) p.233: In Problem 7.27 (iii), "$e^{\mathbf{T}}$" should be "$\mathbf{T}$".

(6) p.233: In Problem 7.27 (iv), "$e^{\alpha\mathbf{T}}e^{\beta\mathbf{T}}$" should be "$e^{\alpha\mathbf{T}} \circ e^{\beta\mathbf{T}}$".

Chapter 8

Section 8.2.

(1) p.244: In Exercise 8.17, "the differential $\mathbf{T}$" should be "the differential of $\mathbf{T}$".

(2) p.245: In line 12, "f" should be boldfaced "$\mathbf{f}$".

(3) p.246: In Exercise 8.21, "for all $\mathbf{a} \in \mathbb{R}^n$" should be "for all $\mathbf{h} \in \mathbb{R}^n$".

Section 8.3.

(1) p.247: In the end of the proof of Theorem 8.23, "the boldfaced $\mathbf{0}$" should be "0".

Section 8.4.

(1) p.250: In line 9, "f" should be "$\mathbf{f}$".

(2) p.251: In the proof of Theorem 8.32, "$\mathbf{v} = \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})$" should be "$\mathbf{a} = \mathbf{x}$, $\mathbf{b} = \mathbf{y}$, and $\mathbf{v} = \mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{x})$".

Section 8.5.

(1) p.257: In line 4, "$\mathbf{g} \in C^1(E)$" should be "$\mathbf{g} \in C^1(E; \mathbb{R}^n)$".

Section 8.6.

(1) p.260: In lines 7 and 13, The period "." should be removed.

(2) p.261: In line 5, The box "□" should be moved to the end of the proof.

Section 8.7.

(1) p.264: In the statement of Theorem 8.55, The letter "s" should be removed from "$g_1(\mathbf{a}) = \cdots = g_m(\mathbf{a}) = 0$s".

(2) p.265: In line 15, "$\mathbf{0} = \nabla\phi(\mathbf{t}_0)$" should be "$O = D\phi(\mathbf{t}_0)$", because they are matrices.

(3) p.265: In line 17, "$\mathbf{0}$" should be the matrix "O".

Section 8.8.

(1) p.266: In Problem 8.4, "$\psi(k\mathbf{x})$" should be "$\psi\left(\frac{\mathbf{x}}{k}\right)$".

(2) p.267: In Problem 8.6 (iii), "harmonic" should be "is harmonic".

(3) p.269: In Problem 8.15 (ii), (iii): "Q" should be "P" and "q_i" should be "p_i".

Chapter 9

Section 9.3.

(1) p.282: The first part of the proof of Theorem 9.24 can be much simplified because the sets S_δ and S'_δ need not to be introduced.

Section 9.4.

(1) p.288: In Example 9.36, "$\mathbf{x}$" should be "(x, y)".

Section 9.5.

(1) p.291: In line 3, "$d\mathbf{g}(\mathbf{x})$" should be "$d\mathbf{g}(\mathbf{x}; \cdot)$".

Section 9.6.

(1) p.298: In line 1, "$\mathbf{g}^{-1} \in C^1(\mathbf{g}(E^\circ; \mathbb{R}^n))$" should be "$\mathbf{g}^{-1} \in C^1(\mathbf{g}(E^\circ); \mathbb{R}^n)$".

Section 9.7.

(1) p.303: In line 14, "2^{m+2}" should be "2^{2m+1}".

(2) p.304: In line 6, "$x^2 + y^2$" should be "$u^2 + v^2$".

(3) p.309: In line 4, "a closed subset" should be "a compact subset". In the end of Example 9.58, we should add the period ".".

Section 9.8.

(1) p.314: In Problem 9.14 (ii), "2π" should be "$2\pi\xi$".

(2) p.315: In Problem 9.16 (i), "$\overline{B_k(\mathbf{0})}$" should be "$\overline{B_{1/k}(\mathbf{0})}$".

(3) p.317: In Problem 9.20 (iii), "$(-2\pi xi)^\alpha$" should be "$(-2\pi i\mathbf{x})^\alpha$".

Hints for Selected Problems

(1) p.322: In Hint for Problem 3.13, the power "p" in the right hand side should be "$p - 1$".

(2) p.324: In Hint for Problem 7.7, "$\mathbb{R}$" should be "$\mathbb{R}^m$".